WJEC CBAC

GCE AS/A level

0978/01

MATHEMATICS – FP2
Further Pure Mathematics

P.M. WEDNESDAY, 18 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S14-0978-01)

1. The function f is defined by

$$f(x) = \frac{x^2 + 1}{x\left(x^2 + 2\right)}.$$

(a) Determine whether f is even, odd or neither even nor odd. [3]

(b) Express $f(x)$ in partial fractions. [4]

2. Using the substitution $u = \sin^2 x$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{4 - \sin^4 x}}\,\mathrm{d}x.$$

Give your answer in the form $\frac{\pi}{k}$, where k is a positive integer. [5]

3. The function f is defined by

$$f(x) = \mathrm{e}^{2x} \qquad \text{for } x < 0,$$

$$f(x) = (x + 1)^2 \qquad \text{for } x \geqslant 0.$$

Determine whether or not

(a) f is continuous when $x = 0$, [3]

(b) the derivative f' is continuous when $x = 0$. [3]

4. The complex number z is given by $1 + \mathrm{i}\sqrt{3}$.

(a) Find the modulus and the argument of z. [2]

(b) Find the three cube roots of z, giving your answers in the form $x + \mathrm{i}y$ with x and y correct to three decimal places. [6]

5. Find the general solution to the equation

$$\sin\theta + \sin 5\theta = \cos 2\theta.$$ [8]

6. Using de Moivre's Theorem, show that for $\sin\theta \neq 0$,

$$\frac{\sin 6\theta}{\sin\theta} = a\cos^5\theta + b\cos^3\theta + c\cos\theta,$$

where a, b, c are constants whose values are to be determined.

Hence determine the limiting value of $\dfrac{\sin 6\theta}{\sin\theta}$ as θ tends to π. [8]

7. The ellipse *E* has equation

$$4x^2 + 9y^2 = 36.$$

(a) Find

(i) the eccentricity,

(ii) the coordinates of the foci. [4]

(b) (i) Show that the point *P*$(3\cos\theta, 2\sin\theta)$ lies on *E* for all values of θ.

(ii) Show that the equation of the tangent to *E* at *P* is

$$3y\sin\theta + 2x\cos\theta = 6.$$

(iii) This tangent meets the x-axis at *R* and the y-axis at *S*. The midpoint of *RS* is denoted by *M*. Determine the equation of the locus of *M* as θ varies. [11]

8. The function f is defined by

$$f(x) = \frac{(x+4)(x-2)}{(x-4)}.$$

(a) Write down the coordinates of the points of intersection of the graph of f and the coordinate axes. [1]

(b) Determine the equation of

(i) the vertical asymptote on the graph of f,

(ii) the asymptote that is not parallel to a coordinate axis. [4]

(c) Find the coordinates of the stationary points on the graph of f. [4]

(d) Sketch the graph of f and its asymptotes. [3]

(e) The set $S = [-7, 3]$. Determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

END OF PAPER